Creating clear designs: a graph-based algorithm and a catalog of clear compromise plans

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Abstract

A graph-based algorithm is proposed for creating regular fractional factorial designs with 2-level factors such that a pre-specified set of 2-factor interactions is clear of aliasing with any main effects or two-factor interactions (clear design). The “clear interactions graphs” used in the algorithm are unique for each design and different in nature from the well-known Taguchi linear graphs. Based on published catalogs of 2-level fractional factorials, enhanced by these graphs, a search algorithm finds an appropriate clear design or declares its non-existence. The approach is applied to creation of a catalog of minimum aberration clear compromise plans, which is also of interest in its own right. Supplementary materials are available for this article. Go to the publisher’s online edition of *IIE Transactions* for additional considerations on run times and implementation of the algorithm for larger designs.

Key words: clear 2-factor interactions, compromise plan, design of experiment, linear graph, clear interactions graph

1. Introduction

In designed industrial experiments, 2-level fractional factorial plans play an important role, since they are both parsimonious in the number of runs and – if designed well – allow estimation of effects of interest. In regular 2-level fractional factorial designs, a small number of runs is achieved by starting from a full factorial in a few, say $m–k$, factors and assigning k additional factors to interaction columns in this design. This generates complete confounding: The 2^m effects of the full model with a constant, i.e. the main effects and all interactions up to order m, can only be estimated as 2^{m-k} sums of 2^k effects each. Consequently, each effect is completely confounded with 2^k-1 other effects. It is often true that interaction effects of more than two factors can be neglected, so that it is not considered problematic if an interaction among three or more factors is confounded with an effect of interest. Thus, it is customary to call effects "clear", if they are not confounded with main effects or 2-factor interactions (2fis) (cf. e.g. Wu and Chen 1992).

There are many ways of assigning m factors to 2^{m-k} runs. The k factors to be added to the full factorial in $m-k$ factors generate 2^k-1 words (groups of factors whose interaction is completely aliased with the overall mean). The length of the shortest word is called the resolution of the design and is denoted as a roman numeral: Resolution V designs do not confound any main effects or 2fis with each other; in resolution IV designs, 2fis can be confounded with each other; resolution III designs even confound main effects with 2fis. Designs of the same resolution are compared using the minimum aberration (MA) criterion, which makes sure that the number of shortest words is minimal (and successively so for the next shortest words in case of ties). For more detail on these concepts, readers are referred to Mee (2009).

In this article, like in much of the related literature, it is assumed that interactions of order higher than two are negligible. Under this assumption, resolution V designs are generally considered adequate, if 2fis are to be estimated. However, they are often not affordable (16 runs for 5 factors, 32 runs for 6 factors, 64 runs for 7 or 8 factors, 128 runs for 9 to 11 factors, 256 runs for 12 to 17 factors, 512 runs for 18 to 23 factors, 1024 runs for 24 to 33 factors etc.). As an aside, note that there are non-regular fractional factorial plans that allow orthogonal estimation of all main effects and two-factor interactions for up to 15 factors in 128 runs or up to 19 factors in 256 runs (cf. e.g. Mee 2009, Chapter 8.2). These are not covered here.

Various authors (e.g. Addelman 1962, Wu and Chen 1992, Ke and Tang 2003, Wu and Wu 2002, Ke, Tang and Wu 2005) have discussed the possibility of devising resolution IV designs such that a pre-specified set of 2fis – called the requirement set in the sequel – can be estimated. While it is often assumed that some 2fis are negligible (e.g. Addelman 1962, Wu and Chen 1992, many articles on *D*-optimal design), this article will make no such assumption. Instead, the requirement set will always contain all main effects and those 2fis which are *of special interest*, without any assumption regarding which 2fis are active. For example, in a robustness experiment with the purpose to find settings of so-called control factors such that the so-called noise factors have as little impact as possible, interactions between control and noise factors may be of special interest, i.e. the requirement set might consist of all main effects and these 2fis, without necessarily assuming negligibility of other 2fis. All effects from the requirement set are estimable, if they are “clear”, i.e. if they are not confounded with any main effect or 2fi, and a design that keeps a requirement set clear is called a clear design in the sequel. Clear designs can be of resolution IV, because 2fis from outside the requirement set need not be clear.

For the above robustness example, now specifically for two noise factors and seven control factors, assume that one experimenter devises an experiment under the assumption that all interactions except those between noise and control factors are negligible, while another experimenter refuses to make that assumption. Both experimenters will end up with a 32 run experiment; the one with the negligibility assumption can use the MA design, while the other experimenter has to deteriorate aberration in order to keep the requirement set clear. This example experiment will be revisited in Section 3. It is one of the fortunate cases for which the requirement set can be kept clear without increasing the number of runs. In many situations clear designs require more runs than analogous designs with negligibility assumptions for the 2fis outside the requirement set. A detailed comparison of the approaches with and without negligibility assumptions can be found in Grömping (2010a).

This article provides a new “clear interactions graph” (CIG) and an algorithm that uses CIGs to find clear designs. Based on a complete catalog of designs, which also lists the unique CIG for each design, the algorithm is guaranteed to find the best clear design for the requirement set or to confirm non-existence of such a design. A preliminary version of the algorithm has been provided in Grömping (2010b).

Section 2 gives a concise overview of cataloging 2-level fractional factorial designs and introduces clear compromise plans. In Section 3, Taguchi (1988) linear graphs are briefly sketched, and the CIGs are introduced. Section 4 proposes the simple algorithm for finding clear designs based on design catalogs, CIGs, and a subgraph isomorphism search algorithm, while Section 5 presents a catalog of minimum aberration clear compromise designs that has been derived using the algorithm of Section 4. Section 6 concludes the article with final remarks. Chapter 2 of the online supplement provides a case study of implementing the algorithm for 256 run designs (which is borderline feasible at the moment because of resource reasons, cf. also Section 4.3). MA clear compromise plans are tabulated in the appendix.

2. Two-level fractional factorials

2.1. Regular 2-level fractional factorial designs

The starting point for a regular 2-level fractional factorial design for *m* factors is a full factorial in 2^{m-k} runs for *m-k* factors, the levels for which are denoted as "-1" and "+1". The model matrix of the saturated model for the full factorial is usually denoted in the so-called "Yates order", which is the obvious continuation of the order **1 2 12 3 13 23 123** …, if **1**, **2**, **3** … are the *m-k* base factors of the full factorial. This matrix is called the "Yates matrix" in the following. Its columns consist of "-1" and "+1" entries, such that columns for interaction effects are obtained as products of the respective main effect columns, with base factor **1** a sequence of 2^{m-k-1} times the pairs -1,+1, base factor **2** a sequence of 2^{m-k-2} times the quadruples -1,-1,+1,+1, base factor 3 a sequence of 2^{m-k-3} times the octuple -1,-1,-1,-1,+1,+1,+1,+1, and so forth. Orthogonality of each pair of effects is easily verified by checking that their scalar product is 0.

A 2^{m-k} run regular fractional factorial design in *m* factors can be uniquely defined by providing the Yates matrix column numbers for the *k* additional (generated) factors. Substantial research has been conducted in order to list non-isomorphic regular fractional factorials, where two designs are considered isomorphic, if they can be obtained from each other by switching rows or columns or levels within columns (cf. e.g. Chen, Sun and Wu 1993, Xu 2009, Ryan and Bulutoglu 2010, Shrivastava and Ding 2010). The non-isomorphic regular fractional factorials for *m* factors in 2^{m-k} runs are usually denoted as *m*-*k*.idno with an index number "idno" denoting the different non-isomorphic versions, and lower "idno" expressing better performance on some overall quality criterion, usually the MA criterion. Table 1 in the appendix provides the designs used in this article in terms of Yates matrix column numbers. For readers used to generator notation, generators can be directly inferred from the binary representation of the Yates matrix column numbers, by using the positions of "1"s from the right to indicate which factors interact. For example, the number 12 has the binary representation 1100, i.e. the third and fourth positions from the right are ones; hence column 12 in the Yates order contains the interaction CD of the third and fourth factor.

Another overall quality criterion for resolution IV designs is MaxC2, i.e. maximization of the number of clear 2fis. For designs with few runs, MA and MaxC2 often coincide (cf. also Wu and Wu 2002). For larger designs, however, there are various situations for which MaxC2 designs are much worse than MA designs in terms of aberration (cf. e.g. Block and Mee 2005). It has been argued that the MA criterion is a good surrogate for model robustness criteria (e.g. Cheng, Steinberg and Sun 1999). The author agrees with this view. Non-MA MaxC2 designs sacrifice some of the model robustness by creating stronger aliasing among the remaining 2fis. Therefore, it is recommended to use MA as the general quality criterion, and to consider clear 2fis on an as-needed basis only, i.e. to only require certain specific 2fis to be clear, if there is a particular interest in their estimation. This is exactly the purpose of the CIG-based algorithm presented here.

2.2. Clear compromise plans

For compromise plans, the m experimental factors are divided into the two groups G1 and G2 with m_1 and m_2 factors respectively, $m_1 + m_2 = m$. For example, in a robustness experiment, G1 might contain control factors and G2 noise factors or vice versa. Four classes of compromise plans have been defined, the requirement sets of which contain all 2fis in

Class 1: G1xG1,

Class 2: G1xG1 and G2xG2,

Class 3: G1xG1 and G1xG2 or

Class 4: G1xG2.

The first three classes were introduced by Addelman (1962), the fourth by Sun (1993). Addelman considered estimability of the specified effects under the assumption that all 2fis outside the requirement set are negligible. This approach is not pursued here. Here, interest is in clear compromise plans which have also been investigated by Ke, Tang and Wu (2005).

Ke et al. proved that there are no clear resolution IV compromise plans of class 2. For the other three classes, they provided lower bounds for the number of runs for a given m_1, as well as upper bounds for m_1 for a given number of runs. Furthermore, they provided a small catalog of clear compromise plans in 32 and 64 runs for class 3 that can also be used for classes 1 and 4 and can in some cases be adapted to special needs by moving a factor from group G2 to group G1 or simply by omitting factors (their tables 1 and 2). They supplemented this catalog with a few additional class 4 clear compromise plans (their table 4). Their catalogs do not make any claims w.r.t. quality criteria of the resulting clear compromise plans. Designs that can be obtained from their catalog directly or by moving or deleting the last factor(s) of a group are shown in bold italics in Tables 2-4 of the appendix. It can be seen that almost all their directly cataloged designs are MA, while designs obtained by moving or deleting columns can often be improved upon.

3. Clear interactions graphs

Before introducing the new clear interactions graphs, Taguchi linear graphs are briefly summarized: these well-known graphs (cf. e.g. Taguchi 1988 for an extensive but incomplete listing) indicate maximum estimable models for each design. Each main effect is a vertex of the graph; each edge represents a 2fi that is estimable, if all 2fis not in the graph as well as all higher order interactions are assumed negligible; the graph includes a maximum possible number of effects, i.e. no further edge can be added without deleting an existing edge. There are several – potentially many – linear graphs for any particular design (for example, according to Wu and Chen 1992 there are 1676 non-isomorphic linear graphs for the 32 run MA design in 10 factors), corresponding to differently structured requirement sets. Wu and Chen (1992) mentioned the possibility to show edges that represent clear 2fis by a special line type. Figure 1 shows one possible linear graph for the design 9-4.1. As factors 9 and 5 have edges with all other factors, they could be used for accommodating the two noise factors from the introductory robustness example, if one is prepared to assume negligibility of all 2fis not shown by an edge in the graph.

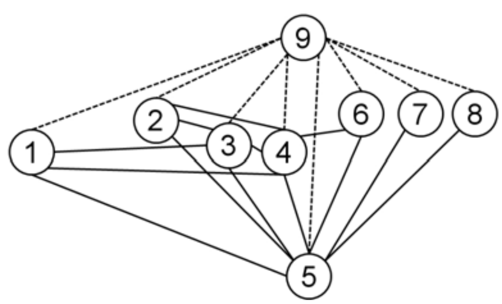

Figure 1: One of many linear graphs for design 9-4.1. Dashed lines denote clear 2fis; all 2fis without edges (e.g. factors 6 and 7) are assumed negligible.
Vertices are labeled with factor numbers (cf. Table 1 in the appendix for corresponding Yates matrix column numbers).

For clear designs, negligibility assumptions for main effects or 2fis are not permitted. This implies that the alias structure with respect to 2fis can be reflected by just one unique graph for each design, the “clear interactions graph” (CIG) as proposed in this article. Again, the factors themselves are the vertices in the graph. The edges are defined by the clear 2fis, i.e. two vertices are connected by an edge, if and only if the 2fi of the respective two factors is clear. Thus, any resolution V design has a complete CIG, i.e. each pair of vertices is connected by an edge, whereas the CIGs of resolution III or IV designs may or may not have edges. Note that the CIG itself does not reveal whether a design is resolution III or IV; usually, resolution III designs should not be considered in CIG applications. It will therefore be assumed throughout this article, that only resolution IV or higher designs are considered.

Figure 2 shows two examples of CIGs: The graph for the MA resolution IV design 9-4.1 in 32 runs and 9 factors indicates that all interactions of the 9th factor with any other factor are clear (the dashed lines in the linear graph for the same design in Figure 1). In the CIG for the second best (in terms of MA) design 9-4.2 all interactions of both the 9th and the 5th factor with each other and all other factors are clear. This implies that the overall MA design 9.4.1 cannot accommodate a clear class 3 or class 4 compromise plan with m_1=2, i.e. cannot accommodate the robustness example from the introduction; the MA design for that purpose is design 9-4.2, assigning the two noise factors to its factors 5 and 9.

Figure 2: Clear interactions graphs for designs 9-4.1 (left) and 9-4.2 (right)
Vertices are labeled with factor numbers (cf. Table 1 in the appendix for corresponding Yates matrix column numbers).

Besides providing a unique CIG for each cataloged design, it is also possible to provide a CIG for an experiment’s requirement set. An intended experiment can be accommodated in a particular design, if its CIG is contained in the design’s CIG, i.e. if there is a mapping of the requirement set graph vertices to the design graph vertices such that all edges in the requirement set CIG are also present in the design's CIG. This comparison can be made by a subgraph isomorphism algorithm, which is the reason why the problem has been cast into graph form in the first place.

As an aside, note that recent work on cataloging fractional factorial designs makes use of bipartite graphs with design rows and design columns (Ryan and Bulotoglu 2010) or factors and defining relations (Shrivastava and Ding 2010) as vertices; a design's structure is characterized by edges in these graphs, which connect pairs of two different type vertices only (e.g. a row and a column); graph isomorphism algorithms are then used for eliminating isomorphic designs. The idea implemented in this article is similar in that it uses algorithms designed for graphs for investigating the structure of experimental designs; however, the graphs are very different, and checks are for subgraph isomorphism rather than graph isomorphism.

4. The proposed algorithm for finding clear designs

As was already mentioned, for any given candidate design, the algorithm employs a subgraph isomorphism search for accommodating the CIG for the requirement set in the design's CIG, if possible. The subgraph isomorphism problem is known to be NP complete. Hence, it is very important to keep the number of candidate designs as small as possible, particularly for larger problems, where "large" relates to the number of structurally different ways the CIG factors could be allocated to the design factors. Efficient use of the algorithm for finding best (e.g. MA) clear designs requires that a complete catalog of designs, ordered from best to worst, is available. The task of finding the best (resolution IV) design with the required 2fis clear is then solved by looping through the cataloged (resolution IV) designs from best to worst, and checking for each design whether the requirement set CIG is contained in the design's CIG. As a result from the algorithm, the experimenter gains the best cataloged solution design or the definite answer that the request cannot be fulfilled within the permitted search designs. In other words, the algorithm is guaranteed to find the best design among the cataloged designs or declares non-existence (within the catalog) of a clear design for the requested requirement set. In the latter case, the experimenter has to increase the number of runs.

It is crucial to have the CIGs stored within the catalog; as each unique CIG can be represented by a two-row matrix with a column for each edge, storing CIGs can be easily accomplished at least for typical application sizes. This opens the possibility to store or easily obtain simple derived design properties like the number of clear 2fis, which can be used for a priori exclusion of unsuitable candidate designs.

4.1. The algorithm and its implementation

The algorithm has been implemented in the free open-source software package **FrF2** (Grömping 2007-2011) within the **R** (R development core team 2011) software environment. It consists of the following four steps:

1. Select the (next) best (criterion: MA) design from a complete catalog of resolution IV designs with the desired number of runs and factors.
2. Does the design have enough clear 2fis for accommodating the requirement set? If yes, continue with next step. If no, go to step 1.
3. Does the design have enough vertices of at least the degrees requested by the requirement set?

   If yes, continue with next step. If no, go to step 1.
4. Apply a subgraph isomorphism search algorithm (algorithm by Cordella et al. 2001 as implemented in R-package **igraph**, cf. Csardi and Nepusz 2006) in order to identify a mapping of experiment factors to design factors such that the requirement set is clear. If such a subgraph is found, the algorithm returns the solution design and this mapping. Otherwise go to Step 1.

Step 1 relies on complete catalogs of designs ordered by the MA criterion available from the literature (Chen, Sun and Wu 1993 with personal communication by Don Sun regarding the resolution IV 64 run designs; Xu 2009 and the supplementary website for resolution IV designs with 128 runs and up to 24 factors). For implementation of the algorithm in the software **FrF2**, the published catalogs have offline been enhanced by their CIGs and by some calculated criteria based on the CIGs, most important the number of clear 2fis in support of step 2. (As the complete catalog for 128 run designs is very large and would inhibit usability of the software, it is provided in the separate **R** package **FrF2.catlg128** (Grömping 2010-2011) for occasional loading when needed, while the software itself only contains a selection of particularly promising 128 run designs.) Obviously, whenever a complete catalog of designs is available for the search process, the algorithm will either return the best possible design – after successful identification of a subgraph mapping between the experiment graph and the design graph – or will exhaust the catalog without finding a design which implies that the requirement set cannot be accommodated within the cataloged designs.

In addition to implementation within the software for up to 128 runs, an exemplary 256 run catalog of the best designs (up to 9 words of length 4) for 18 factors has been created and searched for clear compromise plans, based on work by Ryan and Bulutoglu (2010). This branch of work was instigated by a referee and serves as an investigation into scalability of the algorithm. More MA clear compromise plans were included in the appendix as a result of this extra computational effort described further in chapter 2 of the online supplement.

Note that the algorithm proposed here is quite similar to a proposal made by Wu and Chen (1992) on the basis of linear graphs (i.e. making negligibility assumptions for effects outside the requirement set). Wu and Chen also proposed using a subgraph isomorphism search; an individual subgraph mapping run of their algorithm is in principle identical to step 4 of the above algorithm. As linear graphs are not unique and there can be many such graphs for any moderately-sized design (e.g. 1676 graphs for the MA design in 32 runs for 10 factors, as mentioned in Section 3), their proposal is very resource-intensive and has not been widely applied. The main resource advantage of the CIG-based algorithm consists in uniqueness of the CIG for each design; for example, for 32 runs with 10 factors the algorithm has to search just one CIG for the MA design and at most a total of 27 CIGs for checking any resolution IV design in 32 runs for 10 factors. Nevertheless, the CIG-based algorithm, although often fast for the most typical sizes of experimental situations, is also quite resource-intensive for larger designs, especially with unfortunate requirement sets (cf. Section 4.3).

4.2. A detailed application

The algorithm is now illustrated in detail, using the above robustness example. In the following, denote the seven control factors as C1 to C7, the two noise factors as N1 and N2. The CIG for the requirement set is then shown in Figure 3.

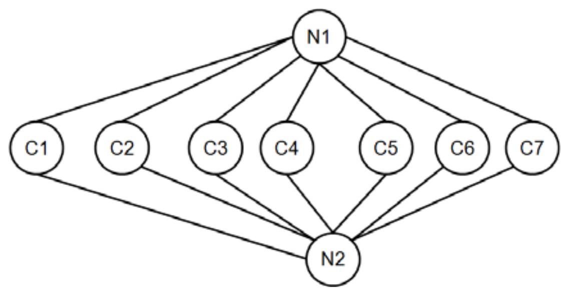

Figure 3: Requirement set CIG for the robustness example

The task of assigning the experimental factors to the appropriate factor numbers of a suitable design now consists in finding the best possible design in the required number of runs for which the graph from Figure 3 is contained in the design's CIG. In this fairly simple example, it is immediately visible by comparing Figures 2 and 3 that the requirement set CIG cannot be accommodated in the CIG for design 9-4.1 but can be accommodated in the CIG for design 9-4.2. This is now also derived by using the algorithm.

Example continued: The design has 9 factors, resolution IV requires 32 runs. The requirement set CIG has 9 vertices and 14 edges; two vertices have seven edges each (degree 7), while the other 7 vertices have degree 2.

Step 1: Design 9-4.1 is the best (=MA) design for the situation at hand.

Step 2: Design 9-4.1 has too few edges (8 clear 2fis = 8 edges only, 14 needed) → loop back to Step 1.

Step 1: Design 9-4.2 is the next best design for the situation at hand.

Step 2: Design 9-4.2 has enough edges (15 clear 2fis = 15 edges).

Step 3: Design 9-4.2 has two vertices with degree 8 and seven vertices with degree 2 (cf. Figure 2) and is thus a candidate worthy of entering Step 4.

Step 4: The subgraph isomorphism search matches experimental factors N1 and N2 to design factors 5 and 9, experimental factors C1 to C7 to design factors 1,2,3,4,6,7,8.

4.3. Resource considerations

As was mentioned before, the subgraph isomorphism problem is known to be NP complete. Even knowing which design will be able to accommodate the requirement set, the search for an adequate mapping (i.e., step 4 of the algorithm in the only iteration) may take a long time in unfortunate situations. For example, searching for how to accommodate the requirement set of a class 3 compromise plan with 17 factors and m_1=3 within the 128 run design 17-10.2407 took about 1.5 seconds when using the first three factors for G1, but – on the same computer under comparable circumstances – more than 48 hours (aborted without finishing) when using the last three factors for G1. As examples for different structures, first and last factors as G1 have been compared for searches for MA class 3 clear compromise plans in 64 or 128 runs – run times were often similar, but sometimes differed strongly (cf. Chapter 1 of the online supplement for more detail). This clearly exemplifies the existence of structurally more or less fortunate requirement sets; a general law indicating which type of requirement set can be accommodated faster for the cataloged designs has not been found; knowing about the structure dependence, it may be worth reformulating a requirement set, given a certain formulation did not deliver a result in reasonable time. The actual optimization for the tables in the appendix was conducted with the first m_1 factors being in G1 and was reasonably fast (for up to 128 runs).

When searching for clear class 3 compromise plans, prefiltering designs with algorithm steps 2 and 3 is very successful, which implies that having to search many designs unsuccessfully until the successful design is encountered is not too detrimental to the run time: For example, finding the mapping for the 128 run design 22-15.118181 for the MA clear class 3 compromise plan with m_1=2 factors within a reduced catalog with this design alone took between 2 and 2.1 seconds, whereas the same search took about 20 seconds within the complete catalog of designs, even though more than 100000 designs had to be identified as inappropriate in the latter case. For class 1 and class 4 compromise plans, pre-filtering is less successful than for class 3 compromise plans because of their less pronounced structure. This is particularly so for class 1. It might help to store the maximum clique sizes with the stored CIGs; however, determining these is another resource intensive task that has not been attacked yet.

Wu and Chen (1992) proposed that researchers should manually compare their requirement set to cataloged graphs rather than using automated processes. Chapter 2 of the online supplement investigates feasibility of identifying clear compromise plans in a catalog of 256 run designs with 18 factors. There, it also turned out that some situations can be addressed by visual inspection that would not have been resolved in adequate time by the algorithm. A combination of using the algorithm for an automatic search with a manual approach based on graph visualization tools turned out to be the best solution for some cases.

5. Complete catalog of smallest MA clear compromise designs

Previously-published catalogs of compromise plans (Addelman 1962 assuming negligibility of 2fis outside the requirement set, Ke et al. 2005 for clear designs) gave a small selection of designs for maximum values of m_1, together with instructions how to obtain further designs from these. Here, the CIG-based algorithm of the previous section has been used for creating complete MA catalogs of clear compromise plans for designs with up to 24 factors in 128 runs (cf. appendix). Ke et al.'s (2005) bounds have been used for limiting the search to possible candidate designs. Additional designs in 256 runs have been included into the tables; noninclusion of a 256 run design does not mean non-existence (cf. also Chapter 2 of the online supplement).

The catalog is presented in four tables in the appendix. Table 1 holds all base designs with their respective Yates matrix columns. Tables 2, 3 and 4 provide the complete listings of resolution IV clear compromise designs of the three classes for which such designs exist. These tables indicate which factors of the respective base design belong to G1; Table 1 can be used for translating the column numbers from Tables 2 to 4 to Yates matrix column numbers.

5.1. Usage examples

Table 3 shows that the smallest MA 16 factor clear compromise plan of class 3 with m_1=2 can be obtained from the design 16-10.45 in $2^{16-10} = 64$ runs using its columns 6 and 16 for G1. According to Table 1, these correspond to Yates matrix columns 32 and 60 for the G1 factors and Yates matrix columns 1 2 4 8 16 7 11 13 14 19 21 22 25 26 for G2. The design entry is set in bold italics, which indicates that an isomorphic design can also be obtained from Ke et al. (2005) by omitting the last G2 factor. The design has 77 words of length 4, which leads to quite heavy confounding (14 model matrix columns hold 6 2fis each, one holds 7 2fis). If a larger design can be afforded and is desired, the footnote to Table 3 indicates that the 128 run design 16-9.2 could be used with G1 columns 4 and 5, which corresponds to Yates matrix columns 8 and 16 for G1 and the remaining Yates matrix columns of the design for G2.

Let us now consider the analogous smallest MA class 1 clear compromise plan with 16 factors and m_1=3: according to Table 2 the MA clear design is based on the base design 16-10.8 and uses its columns 6, 13 and 16 for G1, which translates into Yates matrix columns 32, 25 and 63. This design has better aberration than the one obtainable from Ke et al. (2005). Nevertheless, it is still heavily confounded regarding some 2fis. If a 128 run design is desired for reducing the degree of confounding, the overall MA design 16-9.1 can be used, as it is listed further to the right in the table row for 16 factor designs: an MA clear class 1 compromise design in 128 runs for 16 factors with m_1=3 can be obtained from 16-9.1 using its columns 2, 3 and 5 for G1 (i.e. moving one column from G1 to G2 from the design with m_1=4). Within Tables 2 and 3, it is always permitted to move factors from G1 to G2, as was also stated by Ke et al. (2005) for their corresponding tables 1 and 2. Beware that this is not true for Table 4, where for example the design for 10 factors with m_1=4 can be based on the overall MA design 10-4.1, while m_1=3 requires using the design 10-4.3, which has worse aberration (cf. also the following section).

5.2. Observations regarding MA clear designs

The smallest run resolution IV MA designs in most of the tabulated cases require half the run size of a resolution V design. In some cases, designs with only a quarter of the runs of a resolution V design can also be used. For the latter cases, it may sometimes be desirable to double the run size for reducing the severity of confounding. In most such cases, the larger run size MA clear design can be obtained from the overall MA design in the doubled run size, e.g. from 16-9.1 in the class 1 example from the previous section. Often, this design is listed further to the right in Tables 2 and 3, in which case the MA column allocation for it can be obtained by moving one or more G1 column(s) to G2, as was the case for the class 1 example above. Where a larger MA clear design cannot be obtained in this way (some cases for Table 3, and all cases for Table 4), footnotes indicate how to allocate G1 factors, like in the class 3 example of the previous section.

For clear class 4 compromise plans, it is not permitted to move factors between G1 and G2, which is due to the absence of requirements regarding estimability of 2fis within G1. Consequently, there is no monotonicity in terms of m_1: for example, the overall MA design in 10 factors in 64 runs (10-4.1) can accommodate a clear class 4 compromise plan for m_1=2 or m_1=4, but not for m_1=3: The design has two non-overlapping words of length 4 (positions 1,2,3,7 and positions 5,6,8,9). Choosing all m_1=4 factors for G1 from the same 4-letter word, the design can be used as a clear compromise plan of class 4 for m_1=4, since all confounding is within G1 and within G2 only. When omitting one of these factors from G1, its interaction with the other three becomes important, and the design is not a clear class 4 compromise design for m_1=3. A G1 with m_1=2 factors can again be accommodated within this design, because there are two factors (positions 4 and 10) that do not occur in the two 4-letter words and thus have all their 2fis clear.

The designs that can also be obtained from Ke et al. (2005) have been set in bold italics in Tables 2 to 4. It can be seen that the larger MA clear designs cataloged here are in most cases better than those obtainable from Ke et al. The difference in aberration can sometimes be large, e.g. for a class 1 design with 12 factors and m_1=2, where the Ke et al. design obtained from their 17 run class 3 design by omitting the last 5 G2 columns would result in 18 words of length 4, as opposed to only six such words in the design cataloged here. For other cases, the difference is slight, for example for the class 3 design in 13 factors with m_1=2, where the design obtainable from the Ke et al. instructions has 26 words of length 4 as compared to 25 such words for the MA clear design.

Class 3 compromise plans are also class 4. As the class 3 requirement set is larger, the MA clear class 3 compromise plan is of course not necessarily MA for class 4. However, class 3 and class 4 MA clear compromise plans often coincide. Deviations occur in particular for relatively large values of m_1, because – as has been discussed above – there may be designs with confounded interactions within both G1 and G2 but clear interactions between groups. Class 3 compromise plans are also class 1. Alternatively, one can obtain a class 1 compromise plan with m_1 increased by one vs. a corresponding class 3 plan by moving one factor from G2 to G1. A comparison of Tables 2 and 3 shows that MA clear class 1 compromise plans can often achieve better aberration than the corresponding class 3 plans.

6. Final remarks

Clear interactions graphs (CIGs) have been introduced, and an algorithm based on complete catalogs of CIGs has been proposed that is guaranteed to find the best existing clear design among the cataloged designs. Section 4 provided details on the algorithm and described an implementation in the open-source software package **FrF2** (Grömping 2007-2011). This algorithm has been used for creating a complete catalog of smallest MA clear compromise plans with up to 128 runs and 24 factors; some additional MA clear compromise plans in 256 runs have also been included. This work serves as a demonstration of the usefulness of the CIGs and the CIG-based algorithm. On the other hand, the work on the catalog has also been used to improve the software: those base designs that have shown up as yielding MA clear compromise designs should be generally useful for finding clear designs, even if no perfect compromise design is sought. The 128 run clear compromise plans from the catalog in Tables 2 to 4 are based on 68 different base designs, 30 of which were not originally part of the selection of 128 run designs included in R-package **FrF2**. They have been added to the catalog implemented directly in **FrF2**, which should also improve the chances for automatically finding better designs for general requirement sets. At the very least, the thus-enhanced software will automatically find all MA clear compromise designs cataloged in this article, without loading the additional complete catalog of 128 run resolution IV designs.

Apart from the CIGs and the proposed algorithm, the catalog of clear compromise plans is of interest in its own right, as there are practical situations for which the experimental factors naturally fall into two groups with a certain pattern of 2fis being of interest. For example, in the robustness scenario mentioned in the introduction, control factors and noise factors are a natural choice for the two groups G1 and G2. The interactions between these two groups will be particularly interesting. Nevertheless, one may not be willing to assume negligibility of other interactions. In this case, a class 4 clear compromise design might be adequate. One may also want to estimate interactions among control factors, which will imply a class 3 clear compromise design, whenever one is not willing to assume negligibility of the 2fis among noise factors. Many other such situations are conceivable. Therefore, the catalog of MA clear compromise designs can be quite useful for practitioners.

Finally, note that research on large designs is currently quite active (e.g. Block and Mee 2005; Sanchez and Sanchez 2005; Xu 2009; Ryan and Bulutoglu 2010; Shrivastava and Ding 2010). Extension of the CIG-based algorithm to larger run sizes is nevertheless limited, as was also seen when running the algorithm for searches among the 256 run designs in 18 factors: as the number of resolution IV designs increases dramatically with increasing run size, it will not be feasible to search complete catalogs of large resolution IV designs without prohibitive computing efforts (as of today's computing power). It will however be useful to pre-select large designs with promising alias structures, e.g., many clear 2fis, from newly-found catalogs, which can support many experimental projects with improved designs. In situations for which run size is not crucial, like some computer experiments, resolution V designs may be the answer to the search for clear 2fis: the work by Sanchez and Sanchez 2005 allows to generate very large resolution V designs for up to 120 factors; these are guaranteed to keep main effects and 2fis estimable as long as higher order effects can be considered negligible; however, they are not generally minimum aberration.

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A referee prompted investigation of the algorithm's resource requirements. This led to inclusion of Step 3 into the algorithm, which improves performance for many situations.

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Biography

Ulrike Grömping is a Professor at the Department of Mathematics – Physics – Chemistry at Beuth University of Applied Sciences Berlin, and the Vice Dean of the department. She received her Diploma in Statistics and her PhD from the University of Dortmund, where she also was a lecturer and research assistant. Prior to her current position, she was an applied statistician in environmental epidemiology and in the automotive industry. Her academic and practical expertise includes statistical methods for observational studies and design and analysis of industrial experiments. She is involved in implementing Design of Experiments functionality in the free open source software environment **R**, and she serves on the editorial board of the *Journal of Statistical Software*.

Appendix: Tables of catalogs

Table 1: Resolution IV regular[i] base designs used in at least one clear compromise design
(from the catalogs by Chen, Sun and Wu 1993 or Xu 2009, personal communications by Don Sun (64 runs) and Kenneth Ryan (256 runs, 18 factors) and the web supplement to Xu 2009)

Design *m*-*k*.no.	Runs 2^{m-k}	Column numbers of factors 1 to *m* in Yates matrix[ii]
7-2.1	32	1 2 4 8 16 7 27
8-3.1	32	1 2 4 8 16 7 11 29
9-4.1	32	1 2 4 8 16 7 11 19 29
9-4.2	32	1 2 4 8 16 7 11 13 30
9-3.1	64	1 2 4 8 16 32 7 27 45
10-4.1	64	1 2 4 8 16 32 7 27 43 53
10-4.3	64	1 2 4 8 16 32 7 11 29 51
11-5.1	64	1 2 4 8 16 32 7 11 29 45 51
11-5.4	64	1 2 4 8 16 32 7 11 21 46 56
11-5.6	64	1 2 4 8 16 32 7 11 19 29 62
12-6.1	64	1 2 4 8 16 32 7 11 29 45 51 62
12-6.2	64	1 2 4 8 16 32 7 11 21 46 54 56
12-6.4	64	1 2 4 8 16 32 7 11 21 41 54 56
12-6.23	64	1 2 4 8 16 32 7 11 21 25 31 45
12-5.1	128	1 2 4 8 16 32 64 31 103 43 85 121
13-7.1	64	1 2 4 8 16 32 7 11 21 25 38 58 60
13-7.3	64	1 2 4 8 16 32 7 11 19 29 37 59 62
13-7.6	64	1 2 4 8 16 32 7 11 19 30 37 41 52
13-7.34	64	1 2 4 8 16 32 7 11 13 19 21 25 46
13-6.1	128	1 2 4 8 16 32 64 31 103 43 85 44 86
13-6.6	128	1 2 4 8 16 32 64 31 103 43 85 44 89
14-8.1	64	1 2 4 8 16 32 7 11 19 30 37 41 49 60
14-8.4	64	1 2 4 8 16 32 7 11 19 30 37 41 52 56
14-8.7	64	1 2 4 8 16 32 7 11 19 29 37 41 47 49
14-8.40	64	1 2 4 8 16 32 7 11 13 14 19 21 25 54
14-7.1	128	1 2 4 8 16 32 64 31 103 43 85 46 61 114
14-7.3	128	1 2 4 8 16 32 64 31 103 43 85 46 61 67
14-7.5	128	1 2 4 8 16 32 64 31 103 43 85 44 86 13
14-7.14	128	1 2 4 8 16 32 64 31 103 43 49 74 62 109
14-7.71	128	1 2 4 8 16 32 64 31 103 43 85 121 13 14
14-7.94	128	1 2 4 8 16 32 64 31 103 43 85 44 89 113
15-9.3	64	1 2 4 8 16 32 7 11 19 29 37 41 47 49 55
15-9.9	64	1 2 4 8 16 32 7 11 13 19 21 25 35 37 63
15-9.40	64	1 2 4 8 16 32 7 11 13 14 19 21 22 25 58
15-8.1	128	1 2 4 8 16 32 64 31 103 43 85 46 61 114 67
15-8.3	128	1 2 4 8 16 32 64 31 103 43 85 46 61 114 13
15-8.10	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 55
15-8.34	128	1 2 4 8 16 32 64 31 103 43 85 44 86 13 97
15-8.78	128	1 2 4 8 16 32 64 31 103 43 49 78 45 62 105
15-8.150	128	1 2 4 8 16 32 64 31 103 43 85 44 82 57 113
15-8.423	128	1 2 4 8 16 32 64 31 103 43 85 121 13 14 22
15-8.1221	128	1 2 4 8 16 32 64 31 103 43 85 44 89 113 125
16-10.2	64	1 2 4 8 16 32 7 11 19 29 37 41 47 49 55 59
16-10.8	64	1 2 4 8 16 32 7 11 13 14 19 21 25 35 37 63
16-10.45	64	1 2 4 8 16 32 7 11 13 14 19 21 22 25 26 60
16-9.1	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 53 110
16-9.2	128	1 2 4 8 16 32 64 31 103 43 85 46 61 114 67 78
16-9.80	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 55 56
16-9.890	128	1 2 4 8 16 32 64 31 103 43 85 44 82 57 113 89
16-9.1261	128	1 2 4 8 16 32 64 31 103 43 85 46 61 67 70 105
16-9.1413	128	1 2 4 8 16 32 64 31 103 43 85 46 56 88 79 55
16-9.2913	128	1 2 4 8 16 32 64 31 103 43 85 121 13 14 22 19

Table 1, continued

Design *m-k*.no.	Runs 2^{m-k}	Column numbers of factors 1 to *m* in Yates matrix*
16-9.5539	128	1 2 4 8 16 32 64 31 103 43 85 14 22 13 19 26
17-11.2	64	1 2 4 8 16 32 7 11 19 29 37 41 47 49 55 59 62
17-11.7	64	1 2 4 8 16 32 7 11 13 14 19 21 22 25 35 37 63
17-11.6 = 17-11.38[iii]	64	1 2 4 8 16 32 7 11 13 14 19 21 22 25 26 28 63
17-10.1	128	1 2 4 8 16 32 64 31 103 43 85 46 61 114 67 78 116
17-10.1036	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 55 56 79
17-10.2407	128	1 2 4 8 16 32 64 31 103 43 85 44 82 57 113 89 105
17-10.5846	128	1 2 4 8 16 32 64 31 103 43 85 46 61 67 70 105 108
17-10.5924	128	1 2 4 8 16 32 64 31 103 43 85 46 56 88 79 55 104
17-10.9040	128	1 2 4 8 16 32 64 31 103 43 85 121 13 14 22 19 26
17-10.12633	128	1 2 4 8 16 32 64 31 103 43 85 14 22 13 19 26 28
18-11.1	128	1 2 4 8 16 32 64 31 103 43 85 46 61 114 67 78 116 121
18-11.23	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 53 38 79 98
18-11.95	128	1 2 4 8 16 32 64 31 103 43 85 46 61 114 67 78 116 105
18-11.5146	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 55 56 79 104
18-11.6381	128	1 2 4 8 16 32 64 31 103 43 85 44 82 57 113 89 105 123
18-11.14398	128	1 2 4 8 16 32 64 31 103 43 85 46 56 88 79 55 104 112
18-11.18050	128	1 2 4 8 16 32 64 31 103 43 85 121 13 14 22 19 26 28
18-10.1[iv]	256	1 2 4 8 16 32 64 128 27 46 92 105 127 143 179 182 194 213
18-10.4	256	1 2 4 8 16 32 64 128 127 199 217 51 210 60 102 142 171 7
18-10.21016[v]	256	1 2 4 8 16 32 64 128 127 199 217 242 108 15 54 101 163 170
19-12.1	128	1 2 4 8 16 32 64 31 103 43 85 46 61 114 67 78 55 58 86
19-12.2	128	1 2 4 8 16 32 64 31 103 43 85 46 61 114 67 78 55 58 97
19-12.10	128	1 2 4 8 16 32 64 31 103 43 85 44 82 54 56 88 78 123 125
19-12.488	128	1 2 4 8 16 32 64 31 103 43 85 44 86 25 105 58 106 54 114
19-12.9648	128	1 2 4 8 16 32 64 31 103 43 85 44 82 57 113 89 105 123 125
19-12.11319	128	1 2 4 8 16 32 64 31 103 43 81 45 49 118 73 94 56 104 127
19-12.12482	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 55 56 79 104 112
19-12.26381	128	1 2 4 8 16 32 64 31 103 43 45 87 46 88 55 56 79 104 112
19-11.1	256	1 2 4 8 16 32 64 128 27 46 92 105 127 143 179 182 194 213 229
20-13.1	128	1 2 4 8 16 32 64 31 103 43 85 46 61 114 67 78 55 58 86 91
20-13.2	128	1 2 4 8 16 32 64 31 103 43 85 46 61 114 67 78 55 58 97 108
20-13.11	128	1 2 4 8 16 32 64 31 103 43 85 44 82 54 56 88 78 123 125 104
20-13.43452	128	1 2 4 8 16 32 64 31 103 43 81 44 93 21 13 19 49 14 25 28
20-13.47458	128	1 2 4 8 16 32 64 31 103 43 45 87 46 88 55 56 79 104 112 127
20-12.1	256	1 2 4 8 16 32 64 128 27 46 92 105 127 143 179 182 194 213 229 248
21-14.1	128	1 2 4 8 16 32 64 31 103 43 85 44 82 54 56 88 78 123 125 104 25
21-14.4	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 53 78 58 83 97 28 104
21-14.8	128	1 2 4 8 16 32 64 31 103 43 85 44 82 54 56 88 78 123 125 104 113
21-14.68031	128	1 2 4 8 16 32 64 31 103 43 81 44 93 21 13 19 49 14 25 28 61
21-13.1	256	1 2 4 8 16 32 64 128 27 45 46 92 105 127 143 152 179 203 213 214 236
22-15.1	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 53 78 58 83 97 28 104 114
22-15.7	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 53 78 58 83 97 28 104 112
22-15.8509	128	1 2 4 8 16 32 64 31 103 43 49 74 124 7 61 84 13 67 82 37 62 94
22-15.118181	128	1 2 4 8 16 32 64 63 71 25 104 30 41 78 112 15 49 119 86 23 111 97
22-14.1	256	1 2 4 8 16 32 64 128 27 46 77 88 105 127 143 158 164 179 185 201 213 234
23-16.1	128	1 2 4 8 16 32 64 31 103 43 85 44 82 54 56 88 78 123 125 104 25 112 49
23-16.8	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 53 38 58 79 83 124 114 123 106
23-16.5532	128	1 2 4 8 16 32 64 31 103 43 49 74 124 7 94 14 50 121 100 88 112 21 61
23-16.172917	128	1 2 4 8 16 32 64 63 71 25 104 30 41 78 112 15 49 119 86 23 111 97 46
23-15.1	256	1 2 4 8 16 32 64 128 27 43 46 77 88 105 127 143 158 179 185 201 213 234 236
24-17.2	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 53 38 58 79 83 110 124 97 104 114
24-17.4	128	1 2 4 8 16 32 64 31 103 43 85 44 86 88 53 38 58 79 83 124 114 123 106 113
24-17.4552	128	1 2 4 8 16 32 64 31 103 43 49 74 124 7 94 14 50 121 100 88 112 21 61 13
24-17.256531	128	1 2 4 8 16 32 64 63 71 25 104 30 41 78 112 15 49 119 86 23 111 97 46 39
24-16.1	256	1 2 4 8 16 32 64 128 27 46 77 83 84 105 127 143 146 158 165 166 179 185 213 248

Table 2: Catalog of smallest MA class 1 clear compromise designs (no entry[vi]: resolution V needed)[vii,viii]

Cell entries: design and choice of design columns for G1; "+*n*" denotes the columns of the design to the left with column *n* added

Bold face entry: isomorphic design can also be obtained from Ke et al. 2005.

m_1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
117 factors	***7-2.1*** ***1 4***	***7-2.1*** ***+5***	***7-2.1*** ***+7***								
8 factors	***8-3.1*** ***1 5***	***8-3.1*** ***+8***									
9 factors	9-4.1 1 9	***9-4.2*** ***+5***	***9-3.1*** ***1 4 5 6***	***9-3.1*** ***+8***	***9-3.1*** ***+9***						
10 factors	10-4.1 1 4	10-4.1 +5	10-4.1 +10	***10-4.3*** ***1 5 6 9 10***							
11 factors	11-5.1 1 5	11-5.1 +11	11-5.4 6 8 9 10								
12 factors[ix]	12-6.1 1 5	12-6.2 6 8 9	12-5.1 1 2 3 4	12-5.1 +5	12-5.1 +6	12-5.1 +7	12-5.1 +10	12-5.1 +11			
13 factors[ix]	13-7.1 4 6	13-7.6 4 10 13	13-6.1 1 3 5 7	13-6.1 +8	13-6.1 +9	13-6.1 +10	13-6.6 1 2 3 5 7 8 9 10				
14 factors[ix]	14-8.1 1 10	14-8.7 5 10 13	14-7.1 1 2 4 5	14-7.1 +7	14-7.3 1 4 5 8 9 11	14-7.3 4 5 7 8 9 11 12	14-7.94 1 2 3 5 7 8 9 10				
15 factors[ix]	15-9.3 1 10	15-9.9 6 12 15	15-8.1 1 4 5 6	15-8.1 +11	15-8.34 4 5 7 9 10 13	15-8.1221 1 2 3 5 7 8 9	15-8.1221 +10				
16 factors	16-10.2 1 10	16-10.8 6 13 16	16-9.1 2 3 5 8	16-9.2 1 4 5 6 11	16-9.1261 4 5 7 8 9 11						
17 factors	17-11.2 1 10	17-11.7 6 14 17	17-10.1 1 4 5 8	17-10.1 +9	17-10.5846 4 5 7 8 9 11						
18 factors[x,xi]	18-11.1 1 4	18-11.1 +5	18-11.23 2 5 9 10	18-11.95 1 4 5 8 9	18-10.1 4 5 6 13 14	18-10.1 +18	18-10.1 +1	18-10.1 +2	18-10.4 1 4 5 6 7 9 11 13 15 16	18-10.4 +17	?[xii]
19 factors[x,xi]	19-12.1 1 4	19-12.2 1 8 9	19-12.488 1 8 9 12	19-11.1 1 2 4 5 9	19-11.1 +13	19-11.1 +14	?	?	?		
20 factors[x,xi]	20-13.1 1 4	20-13.2 4 5 15	20-12.1 1 2 4 5	20-12.1 +9	?	?	?	?	?		
21 factors[x,xi]	21-14.1 1 2	21-14.4 2 6 17	21-13.1 1 3 4 6	21-13.1 +9	?	?	?	?	?		
22 factors[x,xi]	22-15.1 1 12	22-15.7 2 6 17	22-14.1 1 2 3 7	22-14.1 +9	22-14.1 +10	?	?	?	?		
23 factors[x,xi]	23-16.1 1 2	23-16.8 2 5 9	23-15.1 1 2 8 9	23-15.1 +15	?	?	?	?	?		
24 factors[x,xi]	24-17.2 1 12	24-17.4 2 5 9	24-16.1 5 6 10 22	?	?	?	?	?	?		

Table 3: Catalog of smallest MA class 3 clear compromise designs (no entry[vi]: resolution V needed)[viii,xiii]

Cell entries: design and choice of design columns for G1; "+*n*" denotes the columns of the design to the left with column *n* added

Bold face entry: isomorphic design can also be obtained from Ke et al. 2005.

m_1	1	2	3	4	5	6	7	8	9	10	11
7 factors	***7-2.1*** ***4***	***7-2.1*** ***+5***	***7-2.1*** ***+7***								
8 factors	8-3.1 5	***8-3.1*** ***+8***									
9 factors	9-4.1 9	***9-4.2*** ***5 9***	***9-3.1*** ***4 5 6***	***9-3.1*** ***+8***	***9-3.1*** ***+9***						
10 factors	10-4.1 4	10-4.1 +10	***10-4.3*** ***5 6 9***	***10-4.3*** ***+10***							
11 factors	11-5.1 11	***11-5.6*** ***6 10***	***11-5.6*** ***+11***								
12 factors[ix]	12-6.4 11	12-6.23 6 12	12-5.1 2 3 4	12-5.1 +5	12-5.1 +6	12-5.1 +7	12-5.1 +10	12-5.1 +11			
13 factors[ix]	13-7.3 10	13-7.34 6 13	13-6.1 5 7 8	13-6.1 +9	13-6.1 +10	13-6.6 1 2 5 7 8 9	13-6.6 +10				
14 factors[ix]	14-8.4 10	14-8.40[xiv] 6 14	14-7.5 7 9 10	14-7.71 6 7 10 11	14-7.94 1 2 5 7 8	14-7.94 +9	14-7.94 +10				
15 factors[ix]	15-9.3 10	***15-9.40***[xv] ***6 15***	15-8.150 1 8 9	15-8.423 6 7 10 11	15-8.1221 1 2 5 7 8	15-8.1221 +9	15-8.1221 +10				
16 factors	16-10.2 10	***16-10.45***[xvi] ***6 16***	16-9.890 1 8 9	16-9.2913 6 7 10 11	16-9.5539 6 7 9 10 11						
17 factors	17-11.2 10	***17-11.38***[xvii] ***6 17***	17-10.2407 1 8 9	17-10.9040 6 7 10 11	17-10.12633 6 7 9 10 11						
18 factors[x,xi]	18-11.1 4	18-11.1 +5	18-11.6381 1 8 9	18-11.18050 6 7 10 11	18-10.1 4 5 6 13 14	18-10.1 +18	18-10.4 5 6 9 11 13 15 16	18-10.4 +17	18-10.21016 2 3 5 6 7 8 9 11 12	18-10.21016 +14	18-10.21016 +15
19 factors[x,xi]	19-12.10 1	19-12.9648 1 8	19-12.9648 +9	?	?	?	?	?			
20 factors[xi]	20-13.11 1	20-13.43452 9 10	?	?	?	?	?	?			
21 factors[xi]	21-14.8 1	21-14.68031 9 10	?	?	?	?	?	?			
22 factors[xi]	22-15.8509 10	22-15.118181 2 3	?	?	?	?	?	?			
23 factors[xi]	23-16.5532 10	23-16.172917 2 3	?	?	?	?	?	?			
24 factors[xi]	24-17.4552 10	24-17.256531 2 3	?	?	?	?	?	?			

Table 4: Catalog of smallest MA class 4 clear compromise designs (entry V[vi]: resolution V needed)[viii,xviii]
Cell entries: design and choice of design columns for G1; "+*n*" denotes the columns of the design to the left with column *n* added
Bold face entry: isomorphic design can also be obtained from Ke et al. 2005.

W.l.o.g., G1 is assumed to be the smaller of the two sets G1 and G2. For larger G1, switch roles of G1 and G2.

m_1	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
7 factors	***7-2.1*** ***4***	***7-2.1*** ***+5***	***7-2.1*** ***4 5 7***							
8 factors	***8-3.1*** ***5***	***8-3.1*** ***+8***	V	V						
9 factors	9-4.1 9	***9-4.2***[xix] ***+5***	***9-3.1*** ***4 5 6***	***9-3.1*** ***1 2 3 7***						
10 factors	10-4.1 4	10-4.1 +10	***10-4.3*** ***5 6 9***	10-4.1 1 2 3 7	10-4.1 +10					
11 factors	11-5.1 11	***11-5.6*** ***6 10***	***11-5.6*** ***6 10 11***	***11-5.1*** ***5 6 9 10***	11-5.1 +11					
12 factors[ix]	12-6.4[xx] 11	12-6.23[xx] 6 12	12-5.1 2 3 4	12-5.1 1 8 9 12	12-5.1 +3	***12-6.1***[xx] ***1 2 3 4 7 8***				
13 factors[ix]	13-7.3[xxi] 10	13-7.34[xxi] 6 13	13-6.1 5 7 8	13-6.1 1 2 11 13	13-6.1 +5	13-6.1 +7				
14 factors[ix]	14-8.4[xiv] 10	14-8.40[xiv] 6 14	14-7.5 7 9 10	14-7.1 1 3 10 12	14-7.1 +4	14-7.1 +5	14-7.14 1 3 5 6 8 11 13			
15 factors[ix]	15-9.3[xv] 10	15-9.40[xv] 6 15	15-8.150 1 8 9	15-8.3 7 9 11 14	15-8.10 1 2 10 11 13	15-8.78 1 5 6 8 11 14	15-8.78 +4			
16 factors	16-10.2[xvi] 10	16-10.45[xvi] 6 16	16-9.890 1 8 9	16-9.80 1 2 11 13	16-9.80 +10	16-9.1413 1 2 3 10 11 12	V	V		
17 factors	17-11.2[xvii] 10	17-11.38[xvii] 6 17	17-10.2407 1 8 9	17-10.1036 1 2 11 13	17-10.1036 +10	17-10.5924 1 2 3 10 11 12	V	V		
18 factors[x,xi]	18-11.1 4	18-11.1 +5	18-11.6381 1 8 9	18-11.5146 1 2 11 13	18-11.5146 +10	18-11.14398 1 2 3 10 11 12	18-10.1 1 3 4 5 6 15 16	18-10.1 +13	18-10.1 +14	
19 factors[x,xi]	19-12.10 1	19-12.9648 1 8	19-12.9648 +9	19-12.11319 2 3 10 12	19-12.12482 1 2 10 11 13	19-12.26381 1 2 3 10 11 13	19-11.1 1 3 4 13 14 15 16	19-11.1 1 3 5 6 15 16 18 19	19-11.1 +4	
20 factors[xi]	20-13.11 1	20-13.43452 9 10	?	20-12.1 1 3 15 16	?	20-13.47458 1 2 3 10 11 13	?	20-12.1 1 3 4 13 14 15 16 20	?	?
21 factors[xi]	21-14.8 1	21-14.68031 9 10	?	21-13.1 4 5 8 16	?	?	21-13.1 4 5 6 8 14 16 17 21	?	?	?
22 factors[xi]	22-15.8509 10	22-15.118181 2 3	?	?	?	?	22-14.1 3 6 8 11 13 17 20	22-14.1 +9	22-14.1 +10	?
23 factors[xi]	23-16.5532 10	23-16.172917 2 3	?	?	?	?	?	?	?	?
24 factors[xi]	24-17.4552 10	24-17.256531 2 3	?	?	?	?	?	?	?	?

[i] There are also non-regular resolution V designs, indicated by footnotes in the following tables. These have not been considered for the catalogs.

[ii] Generators can be determined from the binary representation of the Yates matrix column number: A letter of the alphabet is present in the generator if there is a "1" in the respective position from the right. For example, column 8 is 1 0 0 0 in binary code, which corresponds to letter D; column 12 is 1 1 0 0 in binary code, which corresponds to the 2fi CD.

[iii] The design is called 17-11.6 in the Chen, Sun and Wu catalog in the paper, but 17-11.38 in the complete enumeration of 64 run resolution IV designs as obtained from the authors (personal communication with D.X.Sun). Numbering in the paper reflects some trade-off choices by the authors regarding MA and MaxC2 criteria, numbering in the complete listing is strictly in terms of MA.

[iv] Design 18-10.1 is from the Xu and Wu catalog and not from the complete Ryan and Bulutoglu catalog.

[v] The catalog has been sorted according to WLP up to length 6; design 18-10.21016 is within a large group of designs with the same WLP, even after additional sorting w.r.t. the number of words of length 7. It has minimum aberration among the designs with 11 factors with all their 2fis clear.

[vi] Designs with "?" entries require at least 256 runs; the actual run size is unknown (because a graph-enhanced complete catalog of resolution IV 256 run designs is not available). Exception: the class 1 design with 256 runs and 18 factors, cf. footnote xii.

[vii] The actual run size is larger than the Ke et al. (2005) lower bound for the following combinations (numbers of factors with G1 sizes in parentheses):

10 and 11(1), 12(3 to 5), 13(3 to 4), 14(3), 18 to 22(1), 18 (6 to 8), 19 (5 to 7), 20

and 21(4 to 6), 22 and 23 (4 to 5), 24(4). Whenever the lower bound is 256 and a resolution V design is not possible, the actual run size is not known.

[viii] Designs in bold italics can also be obtained from the Ke et al. (2005) article (up to isomorphism).

[ix] For 12 to 15 factors, there is an irregular resolution V design in 128 runs (cf. e.g. Mee 2009, Section 8.2). This can of course be used as well.

[x] For 18 and 19 factors, there is an irregular resolution V design in 256 runs (cf. e.g. Mee 2009, Section 8.2). This can of course be used as well.

[xi] For the 256 run designs for 18 factors, an incompletely sorted catalog of the best (MA) 508177 designs has been checked (many ties that might that have not been sorted w.r.t. numbers of length 7 or longer words); for the larger 256 run designs, only the MA design has been checked.

[xii] No MA plan was found; the MA clear class 3 plan for 11 factors in G1 (7, 42 and 84 words of lengths 4, 5 and 6, resp.) can accommodate a class 1 compromise plan by moving one factor from G2 to G1. There may be a class 1 256 run resolution IV plan with less aberration.

[xiii] The actual run size is larger than the Ke et al. (2005) lower bound for the following combinations (numbers of factors with G1 sizes in parentheses):
8(3), 10 (1 and 5), 11 (1, 4, 5), 12 (3 to 5), 13 (3,4,8), 14 (3 and 8), 15(8), 16 and 17 (6 to 8), 18 to 22 (1), 18 (5 to 7), 19 (4 to 6), 20 and 21 (3 to 5), 22 (3 and 4). Whenever the lower bound is 256 and a resolution V design is not possible, the actual run size is not known.

[xiv] 14-7.1 would do it with its columns 4 and 5 for G1.

[xv] 15-8.1 would do it with its columns 4 and 5 for G1.

[xvi] 16-9.2 would do it with its columns 4 and 5 for G1.

[xvii] 17-10.1 would do it with its columns 4 and 5 for G1.

[xviii] The actual run size is larger than the Ke et al. (2005) lower bound for the following combinations (numbers of factors with G1 sizes in parentheses):

10,11(1), 12(3 to 5), 13(3,4), 14(3), 16 to 19 (7+), 18 to 22(1), 20 to 22(3 to 5), 23 to 24 (3 to 4). Whenever the lower bound is 256 and a resolution V design is not possible, the actual run size is not known (exception: some 18 run resolution IV designs in 256 runs have also been investigated).

[xix] 9-3.1 would do it with its columns 4 and 5 for G1.

[xx] 12-5.1 would do it with its columns 2, 2 3, or 1 3 4 8 9 12 for G1.

[xxi] 13-6.1 would do it with its columns 5 and 7 for G1.